Let $C_1$ be $x^2 + y^2 = ax$, for which $2x + 2yy' = a$
\[ \Rightarrow y' = \frac{a - 2x}{2y} \]

Let $C_2$ be $x^2 + y^2 = by$, for which $2x + 2yy' = by'$
\[ \Rightarrow y' = \frac{-2x}{2y - b} \]

Then $C_1$ meets $C_2$ where $x^2 + y^2 = ax$ and $x^2 + y^2 = by$
\[ \Rightarrow y = \frac{ax}{b} \quad \text{and} \quad x^2 + \left(\frac{ax}{b}\right)^2 = ax \]
\[ \Rightarrow x^2 \left[ 1 + \frac{a^2}{b^2} \right] = ax \quad (\ast) \]

Let's assume that neither $x$ nor $y$ is zero, because we already know that these circles cross orthogonally at the origin (because the coordinate axes are the tangents, and they're orthogonal). Then $x \neq 0$ in $(\ast)$ yields
\[ x \left[ 1 + \frac{a^2}{b^2} \right] = a \quad \Rightarrow \quad x = \frac{ab^2}{a^2 - b^2} \]
\[ \Rightarrow y = \frac{ax}{b} = \frac{a^2 b}{a^2 - b^2} \]

So, at the point of intersection (other than the origin) we have
\[ \frac{y'}{c_1} \frac{y'}{c_2} = \frac{a - \frac{2ab^2}{a^2 + b^2}}{\frac{2a^2 b}{a^2 + b^2}} \cdot \left\{ \frac{-2ab^2}{a^2 + b^2} \right\} \]
\[ = \frac{a (a^2 + b^2) - 2ab^2}{2a^2 b} \cdot \left\{ \frac{-2ab^2}{a^2 + b^2} \right\} \]
\[ = \frac{a (a^2 - b^2)}{2a^2 b} \cdot \frac{-2ab^2}{b (a^2 - b^2)} = -1 \]