1. Find the inverse of the matrix \( A = \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix} \) and use it to solve the equation \( Ax = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \).

2. Let \( A = \begin{bmatrix} 1 & 4 & -2 & 3 & -2 \\ -2 & -8 & 4 & -5 & 7 \\ 3 & 12 & -6 & 9 & -9 \end{bmatrix} \).
   (a) Find the rank of \( A \).
   (b) Find a basis of the null space of \( A \).
   (c) Find a basis of the column space of \( A \).

3. Let \( A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & 1 & 4 \end{bmatrix} \).
   a. Find the eigenvalues of \( A \); for each eigenvalue \( \lambda \), find a basis of the corresponding eigenspace of \( A \).
   b. Is \( A \) diagonalizable? Why?

4. Let \( A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix} \). Find an invertible matrix \( P \) and a diagonal matrix \( D \) such that \( A = PDP^{-1} \).

5. Find the characteristic polynomial of the matrix \( A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 0 \\ 2 & -1 & 2 \end{bmatrix} \).

6. Let \( T: \mathbb{R}^2 \to \mathbb{R}^2 \) be defined by \( T(x) = Ax \), where \( A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \). Find the matrix \([T]_B\) of \( T \) relative to the basis \( B = \{b_1, b_2\} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\} \).

7. A \( 2 \times 2 \) matrix has eigenvalues \( \lambda_1 = 1 \) and \( \lambda_2 = 1/2 \) with corresponding eigenvectors \( v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) and \( v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \). Find a formula for \( A^k \), where \( k \) is a positive integer.

8. True or False?
   (a) If \( A \) is an \( n \times n \) matrix such that the linear transformation \( T(x) = Ax \) is one-to-one, then \( A \) is invertible.
   (b) The null space of a \( 4 \times 5 \) matrix of rank 4 must be trivial; that is, it must equal \( \{0\} \).
   (c) If an \( n \times n \) matrix \( A \) has \( n \) linearly independent eigenvectors, then \( A \) has \( n \) distinct eigenvalues.
   (d) If \( \lambda = 0 \) is an eigenvalue of \( A \), then \( A \) is not invertible.